Transfers in simple homotopy theory

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Abstract: Simple homotopy theory is the study of finite CW-complexes up to elementary collapses and expansions. From its early stages, it was observed that simple homotopy types are deeply connected to K-theory. This connection is realized through Wall's finiteness obstruction for finitely dominated complexes and the Whitehead torsion of homotopy equivalences between finite complexes.

In this talk, following insights of Waldhausen, I will give an ahistorical introduction to simple homotopy theory from the perspective of assembly maps in K-theory and transfers. I will explain how a new viewpoint on assembly maps, grounded in the six functor formalism for sheaves of spectra on locally compact Hausdorff spaces, leads to several applications. These include generalizations of West's theorem on the homotopical finiteness of compact ANRs (1977), Chapman's theorem on the topological invariance of Whitehead torsion (1974), the topological Dwyer–Weiss–Williams index theorem (2003), and a fully coherent functoriality of Becker–Gottlieb transfers. The proof of the latter is very robust and works verbatim in any six-functor formalism, such as étale sheaves or motivic spectra.

This is joint work with Maxime Ramzi and Sebastian Wolf.