Categorical trace techniques for representations of finite groups of Lie type

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Finite groups of Lie type are groups of the form $G(\mathbb{F}_q)$ for reductive algebraic groups G over finite fields, examples include $\mathrm{GL}_n, \mathrm{SL}_n$ or SO_n. They form a very large collection of finite groups and include most finite simple groups (16 of the 18 infinite families according to the classification of finite simple groups) and their representation theory gives a model for the representation theory of all finite simple groups. Exploiting their essentially algebraic nature, Deligne and Lusztig constructed all (complex) irreducible representations in the cohomology of, what are now called, the Deligne–Lusztig varieties. The geometric methods they introduced has been key in settling a number of questions concerning the complex representation theory of these groups. On the other hand, the modular representation theory of finite groups of Lie type is still very mysterious. In the recent years, new ideas, the so called categorical traces, have been introduced and provide a link between geometric representation theory and the representation theory of these finite groups of Lie type. In this talk, I will give an overview of these methods, their results and some (still open) conjectures of Deligne-Lusztig theory.