

Explicit methods for enumerative invariants using Siegel and mock modular forms: The K3 case

Abhiram Kidambi (MPI Leipzig)

A conjecture for nearly two decades on the interface of algebraic geometry, number theory and topological string theory is the OSV conjecture which conjectures a form for the generating function of (certain) rank 0 Donaldson-Thomas invariants for CY threefolds, and this formulation goes hand in hand with understanding wall crossing phenomena. On the automorphic forms side, one often encounters mock modular forms and iterated integrals thereof. To understand the precise relation between mock-modularity, OSV conjecture is an open problem as well.

There are many problems in the same spirit, and we will visit the problem in the case of K3 surfaces and symmetric powers thereof. This case is a particularly nice exercise since the analogue of the OSV conjecture is a bit more restrictive (because there is only one polynomial term in the formal power series expansion, which is needed to compute the enumerative invariants). This allows us to compute things rather explicitly using the theory of mock modular/Jacobi forms. Of particular interest are special points (called attractor points) where the generating function for enumerative invariants follow an OSV-type behaviour.

In this (chalk) talk, I will start with the statement of the problem for this particular case, introduce all the necessary objects (Siegel modular forms, mock modular forms, Jacobi forms) and explain how one can construct the generating function for enumerative invariants at special points (and generic points if time permits).