

Combing a hedgehog over a field

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A classical result in differential topology says that there are no nowhere vanishing vector fields on a 2-sphere. One may ask a similar question in algebraic geometry: does the tangent bundle to a sphere given by the equation $x^2 + y^2 + z^2 = 1$ over some field k have a nowhere vanishing section? Or more generally, when does the tangent bundle on an affine quadric $q = 1$ with q being a homogeneous degree 2 polynomial have a nowhere vanishing section? We give an essentially full answer to this question assuming that the quadric $q = 1$ has a rational point. In particular, the 2-sphere $x^2 + y^2 + z^2 = 1$ over a field k has a nowhere vanishing vector field if and only if -1 is a sum of 4 squares in k . The proof uses a mixture of results from the motivic homotopy theory, Chow-Witt rings and some constructions from the theory of quadratic forms. This is a joint work with Marc Levine.