Motivic homotopy theory beyond $\mathbb{A}^1$-invariance

Ryomei Iwasa (Paris)

The basic question I'd like to address in this talk is the following: How to do homotopy theory in algebraic geometry while keeping the affine line $\mathbb{A}^1$ non-contractible? I will explain that tensor invertibility of the pointed projective line $\mathbb{P}^1$ supplies homotopies between projective bundle sections in a non-trivial but canonical way. This dramatically expands the scope of motivic homotopy theory, and non-$\mathbb{A}^1$-invariant theories such as syntomic cohomology, prismatic cohomology, algebraic K-theory, and topological cyclic homology can be studied from this perspective. In particular, I'll explain that algebraic and Selmer K-theory are described by Snaith-type formulas. Based on joint work with Toni Annala and Marc Hoyois.