Übungsblatt 3

Abgabe am Freitag, 12.07.2019 bis 23 Uhr

Aufgabe 3.1. Prove or disprove:

1. The integral closure of $\mathbb{Z}$ in $\overline{\mathbb{Q}}$ is a Dedekind domain.
2. If ζ is a primitive 5-th root of unity in $\mathbb{C}$ and $K = \mathbb{Q}(\zeta)$, then $1+\zeta$ is an element of $O_K^\times$ and the group generated by $1+\zeta$ in $O_K^\times$ is of finite index.
3. If K is a number field, then there is a finite extension L of K such that, for all ideals $\mathfrak{a}$ of $\mathcal{O}_K$, the ideal $\mathfrak{a}\mathcal{O}_L$ is principal.
4. A Dedekind domain with precisely two maximal ideals is a principal ideal domain.
5. If A is a Dedekind domain and $\mathfrak{a}$ is a nonzero ideal of A, then all ideals of the quotient ring $A/\mathfrak{a}$ are principal.
6. Every ideal of a Dedekind domain can be generated by two elements.
7. An integral domain of dimension at least one in which all nonzero ideals admit a unique factorization into prime ideals is a Dedekind domain.

Aufgabe 3.2. Let K and L be finite Galois extensions of $\mathbb{Q}$. Suppose that there is a prime number p such that p is unramified in K and totally ramified in L. Show that K and L are linearly disjoint.