

## HW 8: Elliptische Kurven I

- Hand in by June 14th 2016.

**Exercise 1.** Let  $P_1 = (1 : 0 : 0)$ ,  $P_2 = (0 : 1 : 0)$ , and  $P_3 = (0 : 0 : 1)$  in  $\mathbb{P}^2$ . Let  $U = \mathbb{P}^2 \setminus \{P_1, P_2, P_3\}$ . Let  $f : U \rightarrow \mathbb{P}^2$  be the morphism defined by

$$f(a_0 : a_1 : a_2) = (a_1 a_2 : a_0 a_2 : a_0 a_1).$$

Show that there is no morphism  $F : \mathbb{P}^2 \rightarrow \mathbb{P}^2$  which extends  $f$ .<sup>1</sup>

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<sup>1</sup>This exercise shows that  $f$  does not extend to a morphism of projective varieties  $X \rightarrow \mathbb{P}^2$  with  $U \subset X$  open and  $X \cong \mathbb{P}^2$ . On the other hand, there are more projective varieties  $X$  which contain  $U$  as an open, e.g., the “blow-up” of  $\mathbb{P}^2$  in  $P_1$ . If we take  $X$  to be a suitable blow-up of  $\mathbb{P}^2$ , then there is an extension of  $f$  to a morphism  $X \rightarrow \mathbb{P}^2$ . Unfortunately, we won’t get to discuss these matters in this course.