HW 6: Elliptische Kurven I

- Hand in by May 31st 2016.

Exercise 1. Let $n \geq 1$ and let PGL_n be the quotient group $\mathrm{GL}_n/(k^\times \cdot \mathrm{id}_n)$ of $n \times n$ matrices modulo scalars. Show that there is an integer $N \geq 1$ such that PGL_n is a closed subset of GL_N. (Conclude that PGL_n is a linear algebraic group.)

Exercise 2. Prove or disprove:

1. A quasi-affine variety is isomorphic to an affine variety.
2. A bijective morphism of affine varieties is an isomorphism of varieties.
3. Let $\mathcal{C}$ be a category. If X and Y are objects of $\mathcal{C}$, then a product of X and Y exists in $\mathcal{C}$.
4. Let K be a field. If A and B are finitely generated k-algebras with no zero divisors, then $A \otimes_K B$ has no zero divisors.