HW 4: Algebraische Geometrie II

Handing in: Hand in by May 17t 2016. Grades are from 1 to 10. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly!

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Is the ideal $I = (x^4 + y^4 - 2x^2y^2 + x^2 - y^2)$ in $\mathbb{R}[x, y]$ a radical ideal in $\mathbb{R}[x, y]$?

Exercise 2. Let X be a Hausdorff topological space. Determine all irreducible subsets of X.

Exercise 3. Let U be a dense open of $\mathbb{A}^1$ which is (abstractly) isomorphic to $\mathbb{A}^1$ as a variety. Show that $U = \mathbb{A}^1$.

Exercise 4. Let f be an irreducible polynomial in $\mathbb{R}[x, y]$. Define $Z(f)(\mathbb{R}) := \{(a, b) \in \mathbb{R}^2 \mid f(a, b) = 0\} \subset \mathbb{A}^2(\mathbb{C})$. Is $Z(f)(\mathbb{R})$ necessarily irreducible (with respect to the subspace topology on $Z(f)(\mathbb{R})$)?