HW 3: Elliptische Kurven I

Handing in: Hand in by May 10th 2016. Grades are from 1 to 10. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly!

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Prove or disprove (by means of a counterexample) the following statements.

1. A connected closed subset of affine space is irreducible (in the Zariski topology).
2. All non-empty open subsets of an irreducible topological space are dense and irreducible.
3. Let k be an algebraically closed field. If one identifies $\mathbb{A}^2(k)$ with $\mathbb{A}^1(k) \times \mathbb{A}^1(k)$, then the Zariski topology on $\mathbb{A}^2(k)$ is the product topology of the Zariski topologies on the two copies of $\mathbb{A}^1(k)$.
4. Let k be a field. The subring R of $k[x, y]$ consisting of all polynomials of the form $f(x) + x \cdot g(x, y)$ is not noetherian.
5. A closed subset X of $\mathbb{A}^n$ is quasi-compact. (Here we follow Bourbaki's terminology, so that **quasi-compact** means that every open cover of X has a finite subcover. Then again, in Bourbaki, a topological space is defined to be **compact** if it is quasi-compact and Hausdorff.)