HW 2: Elliptische Kurven I

Handing in: Hand in by May 3rd 2016. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly!

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Let k be an algebraically closed field. Prove or disprove the following statements.

1. If $Y \subset \mathbb{A}^2$ is the zero set of $y - x^2$ (over k), then the ring $\mathcal{O}(Y)$ is isomorphic to a polynomial ring in one variable.
2. If $Z \subset \mathbb{A}^2$ is the zero set of $xy - 1$, then $\mathcal{O}(Z)$ is isomorphic to a polynomial ring in one variable.
3. The polynomial $f = (y - 2x + 1)y$ in $k[x, y]$ defines a connected algebraic subset of $\mathbb{A}^2$.

Exercise 2. Let Y be the common zero set of the polynomials $x^2 - yz$ and $xz - x$ in $\mathbb{A}^3(k)$, where k is an algebraically closed field. Show that Y is the union of three irreducible components. Describe them and find their prime ideals (in $k[x, y, z]$).