

HW 1: Elliptische Kurven I

Handing in: Hand in by April 26th 2016. Keep a copy of your work. You are strongly encouraged to write your solutions in TeX and hand in a printout. If you write your solutions by hand, write legibly!

The rules of the game: Your solutions to these exercises should convince us that you have a good understanding of what has been discussed so far. This involves more than just getting the correct answers, which, anyway, might be obtained from various internet sources or the brain of a fellow student. You may use everything that has been discussed so far in the course. Give full details for your arguments, with precise references for the results you use. What you hand in should be your own, individual work.

Exercise 1. Prove or disprove (by means of an explicit counterexample) the following statements.

1. If k is an algebraically closed field, then the polynomial $x^2 + y^2 - 1$ in $k[x, y]$ is irreducible.
2. An algebraically closed field k has infinitely many elements.
3. If K and L are fields and there is a morphism of rings $K \rightarrow L$, then K and L have the same characteristic.