HW 10: Elliptische Kurven I

- Hand in by June 28th.

Exercise 1. Let $X = Z(y^2 - x^3)$ in $\mathbb{A}^2$. Let $p = (0,0)$ and $q = (1,1)$.

1. Compute the singular locus of X.
2. Compute the maximal ideals $\mathfrak{m}_p$ and $\mathfrak{m}_q$ of p and q in $\mathcal{O}(X)$, respectively.
3. Compute $\mathfrak{m}_p/\mathfrak{m}_p^2$ and $\mathfrak{m}_q/\mathfrak{m}_q^2$ and their dimensions as k-vector spaces.

Exercise 2. Let X be a quasi-projective irreducible curve.

1. Assume that X is projective. Show that there exist hyperplanes H_1 and H_2 in $\mathbb{P}^n$ such that $H_1 \cap H_2 \cap X = \emptyset$.
2. Assume that X is projective. Show that there exist $U \subset X$ open and $V \subset X$ open such that U is affine, V is affine, and $X = U \cup V$.
3. (Difficult) Show that there exist hypersurfaces $Z_+(f_1)$ and $Z_+(f_2)$ in $\mathbb{P}^n$ such that $X \cap Z_+(f_1) \cap Z_+(f_2) = \emptyset$ and $X \cap D_+(f_i)$ is closed in $D_+(f_i)$ for both i.