Exam: Elliptische Kurven I

- Hand in by August 2nd 2016.

Let k be an algebraically closed field.

Exercise 1. Prove or disprove:

1. If X is a smooth projective irreducible curve of genus one over k, then the automorphism group of X (as a quasi-projective variety) is commutative.
2. The set of isomorphism classes of elliptic curves over $\overline{\mathbb{F}_p}$ is countable.
3. The set of isomorphism classes of elliptic curves over $\mathbb{C}$ is countable.
4. If $d \geq 3$ is an integer and C_d is the cyclic group of order d, then there is a smooth projective irreducible curve X_d such that $C_d \times C_2$ maps injectively to $\mathrm{Aut}(X_d)$.

Exercise 2. For $t \in k \setminus \{0\}$, let E_t be the elliptic curve given by $y^2 = x^3 + t$. Show that, for all t and t' in $k \setminus \{0\}$, we have $E_t \cong E_{t'}$.

Exercise 3. Let $g \geq 1$ be an integer. For a in $\mathbb{C}$, let C_a be the affine curve

$$y^2 = (x-a) \prod_{i=0}^{2g-1} (x-i)$$

in $\mathbb{A}^2 = \mathbb{A}^2(\mathbb{C})$, where x and y are coordinates on $\mathbb{A}^2$. Show that C_a is smooth if and only if

$$a \notin \{0, 1, \ldots, 2g-1\}.$$